DOCUMENT RESUME

ED 183 587 TM 010 106

AUTHOR Stewart, Bruce L.
TITLE Teaching Logic in Grades 7-12: A Literature Review. Critical Thinking Reports Number 1.
INSTITUTION Illinois Univ., Urbana. Bureau of Educational Research.
PUB DATE 78
NOTE 41p.; For related document, see TM 010 107

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Annotated Bibliographies; Cognitive Tests; Critical Thinking; *Deductive Methods; *Effective Teaching; Literature Reviews; *Logic; Secondary Education; Teaching Methods

ABSTRACT

Literature on teaching deductive logic in grades 7 through 12 is reviewed. The fourteen studies included in the review are presented in tabular form. For each study, the following information is given: (1) author, date, student grade; (2) logical principles taught; (3) success claimed for the method; (4) experimental design; (5) criterion measure; (6) strengths of the teaching methods; and (7) weaknesses. In addition, summaries and reviews of the cited studies are appended. (MH)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED183587

U S DEPARTMENT OF HEALTH, EDUCATION & WELFARE
NATIONAL INSTITUTE OF EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

TEACHING LOGIC IN GRADES 7-12: A LITERATURE REVIEW

BY

BRUCE L. STEWART

CRITICAL THINKING REPORTS
NUMBER 1

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Bruce L. Stewart

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

IM010 106

Published by

The Illinois Critical Thinking Project

Bureau of Educational Research
University of Illinois at Urbana-Champaign
Urbana, Illinois
1978

Table of Contents

	Page
General Discussion	1
Summary Table for Studies Cited in this Review . . .	9
References	14
Appendix: Summaries and Reviews of Cited Studies	
Braxton, L. M.	16
Carroll, C. A.	18
Corley, G. J.	20
Diffley, W. J.	22
Ennis, R. H. & Paulus, D. H.	23
Howell, E. N.	26
Hrabi, J. S.	27
Hyram, G. H.	29
Karplus, E. F. & Karplus, R.	30
Miller, E. H.	32
Miller, W. A.	33
Osborne, A. R.	34
Retzer, K. A. & Henderson, K. A.	35
White, E. E.	37

TEACHING LOGIC IN GRADES 7-12: A LITERATURE REVIEW

Bruce L. Stewart[1]
Critical Thinking Project
Bureau of Educational Research
University of Illinois

This paper presents a review of the literature dealing with attempts to teach deductive logic in grades 7-12. Two criteria were used in determining whether a particular study was to be included in this review. These criteria were:

(1) Grade level of students being taught logic; in order to be included, a study had to include teaching of logic to at least some students in grades 7-12. Studies which dealt exclusively with teaching logic to students below the seventh grade level were not included.[2]

(2) The type of "logic" taught; in order to be included in this review, a study must have dealt with an attempt to teach what could be broadly construed as "deductive logic." A study by Linn and Thier (1975) was excluded from this review because their research focused on the Piagetian notion of "compensation" as the criterion measure of logical ability, and the nature of the relationship, if any, between "compensation" and deductive logic is unclear.

Studies were not excluded on the basis of any qualitative judgments of their rigor.

Two major difficulties were involved in locating studies to be included in this review. First, discussions of attempts to teach logic

---

[1] I am indebted to Robert Ennis for his many helpful criticisms and suggestions on earlier versions of this paper.

[2] However, if primary school students can master some logical principles, it seems reasonable to suppose that secondary school students are also capable of mastering these principles, and possibly others in addition.

in secondary school do not appear regularly in the journals. One explanation for this state of affairs might be that in those schools where logic is taught as part of the curriculum (either as a separate subject or as part of some other subject), the question, "Can logic be taught successfully in secondary school?" is presumed answered. Hence, little motivation exists to report the results of a logic teaching program unless such results were outstanding. In contrast, articles which argue for the need for instruction in logic are fairly common. Second, studies of attempts to teach logic are often indexed in the literature under headings which may be related to, but do not mention, deductive logic as such. These headings include "Thinking," "Thinking Skills," "Reasoning," "Critical Thinking," and "Inferential Reasoning" to name a few. Time constraints did not permit a thorough investigation of all of these possibly relevant subject headings. For this reason, this review cannot claim to be comprehensive.

However, the literature search undertaken for this review covers those studies which appear in the Education Index for a period of approximately the last 20 years. In addition, whenever earlier studies were referenced, such referenced studies were located for possible inclusion in this review. Educational Resources Information Center (ERIC) documents were also examined, for the period of approximately the last 10 years.

This review includes studies which report attempts to teach logic both as a part of other programs (for example, mathematics or English) and as a subject in its own right (compare Howell, 1967; Hyram, 1957; and Retzer and Henderson, 1967, for example). The review also includes

studies reporting attempts to teach logic to students from diverse backgrounds (compare Carroll, 1970; Corley, 1959; Diffley, 1971; and Retzer and Henderson, 1967). Caution is required in comparing results of the studies reviewed here since different authors have different notions of "logic" and therefore their conclusions about the "logical ability" or "logical achievement" of students are not necessarily open to direct comparison.

Eleven of the fourteen investigators covered in this review claimed success in teaching logic in secondary school (Braxton, 1973; Diffley, 1971; Ennis and Paulus, 1965; Howell, 1967; Hyram, 1957; Karplus & Karplus, 1970; Miller, 1955; Miller, 1969; Osborne, 1973; Retzer and Henderson, 1967; and White, 1936). These claims were, for the most part, based upon pupil scores on some test of logical reasoning ability, usually administered after a period of instruction in logic (details of the individual experimental designs appear in the Appendix). In some cases, these claims were partly or completely based upon subjective evaluation of pupil classroom performance by the experimenter and/or classroom teacher (Hyram, 1957; Miller, 1969). Some studies also employed more than one criterion measure (Ennis and Paulus, 1965; Howell, 1967; Hrabi, 1967; and White, 1936). In the majority of studies cited here, the criterion measure was designed by the experimenter (Carroll, 1970; Corley, 1959; Ennis and Paulus, 1965; Howell, 1967; Hrabi, 1967; Hyram, 1957; Karplus and Karplus, 1970; Miller, 1955; Retzer and Henderson, 1967; and White, 1936). The criterion measures not developed by the experimenter were: "The Cornell Conditional Reasoning Test, Form X" (Braxton, 1973; Osborne, 1973); "The Paulus Conditional Reasoning Test, Form Z" (Diffley, 1971);

and "The Watson-Glaser Critical Thinking Appraisal, Form YM" (Hrabi, 1967). Further, Miller (1969) uses a "Test of Logical Inference Patterns" whose origin he does not specify; it seems possible that this test is the "Test of Inference Patterns" developed by Howell (1967). Because a number of different criterion measures were used, and inter-correlations are not available, comparisons of achievement in logical reasoning across studies are difficult to make.

The investigators also vary widely in their definition of "logic." Retzer and Henderson (1967) deal primarily with mathematical logical concepts; Hyram (1957) discusses the "nature" of reasoning-related concepts under his definition of "logic"; and Karplus and Karplus (1970) offer only a problem which they claim tests logical ability but fail to specify which logical principle(s) might be involved. Generally, however, experimenters appeared to have some form of deductive logic in mind when they used the term, "logic." Six experimenters explicitly include these conditional reasoning principles in their definition of "logic":[3]

(1) detachment:	if p, then q; p; therefore, q.
(2) full transitivity:	if p, then q; if q, then r; therefore, if p, then r.
(3) particular contraposition (denying the consequent):	if p, then q; not q; therefore, not p.
(4) particular conversion (affirming the consequent--an invalid move):	if p, then q; q; therefore, p.
(5) particular inversion (denying the antecedent--an invalid move):	if p, then q; not p; therefore, not q.

[3] This system of classifying logical principles is taken from (Ennis, 1976).

(Braxton, 1973; Carroll, 1970; Diffley, 1971; Ennis and Paulus, 1965; Howell, 1967; Miller, 1969). The definitions of other experimenters can be reasonably construed to include at least some of the above principles (Corley, 1959; Hrabi, 1967; Miller, 1955; White, 1936).

A further complication in comparing studies lies in the time allotted for the teaching of logic. Some studies which report only moderate success in teaching logic were also those in which the least time was spent on teaching logic (Carroll, 1970; Corley, 1959). Generally, experimenters who spent longer on the teaching of logic (either by instructing the classes themselves or by using regular classroom teachers) claimed the most success in teaching logic, as measured by students' performance on criterion. However, these studies can also be questioned on the grounds of "teaching for the test" since often the experimenter both developed criterion measures and served as the instructor (Corley, 1959; Ennis and Paulus, 1965; Hyram, 1957; Miller, 1955; Miller, 1969; Retzer and Henderson, 1967; and White, 1936).

Another point which emerges from this literature search is that claims of successful teaching of logic to secondary school students were not limited to students from a particular background. For example, Diffley (1971) worked with inner city students; Carroll (1970) worked with students classified as "low achievers in mathematics"; Retzer and Henderson (1967) studied both gifted (I.Q. = 135 and above) and nongifted (I.Q. = 116-128) students; Corley, (1959) studied students with a minimum I.Q. of 92.5. Each of these authors claimed success in teaching at least some logical principles to secondary school students. In addition, four experimenters reported that students learned logic when taught by

their regular teachers (Braxton, 1973; Diffley, 1971; Howell, 1967; Hrabi, 1967). The study by Osborne (1973) suggests that students may learn conditional reasoning principles even when not directly taught if the teacher utilizes conditional statements in normal classroom procedures.

The logic instruction units in some studies explicitly required students to learn to recognize examples of both valid and invalid reasoning (Braxton, 1973; Carroll, 1970; Diffley, 1971; Ennis and Paulus, 1965; Miller, 1969), and the instructional units of other experimenters could be reasonably construed to include this requirement (Corely, 1959; Hrabi, 1967; Hyram, 1957; Miller, 1955). Studies which examined differential mastery of individual logical principles supported the findings of other authors that recognition of valid logical principles was more easily mastered than recognition of invalid logical principles (Braxton, 1973; Carroll, 1970; Diffley, 1971; Ennis and Paulus, 1965; Miller, 1969).[4] Some experimenters also report varying the content of their test items to include examples of concrete, suppositional, and "fantasy" items[5] (Carroll, 1970; Diffley, 1971;

---

[4] Howell's study (1967) suggests that some valid logical principles are more easily mastered than other valid logical principles. It is not clear from the study that students were exposed to both valid and invalid forms during the instructional unit, and hence, this study cannot be used to support the claim that valid forms are more easily recognized than invalid forms. Howell does state, however, that exposure to valid forms is insufficient for students to recognize invalid forms, which suggests that the instructional unit in this study dealt exclusively with valid forms.

[5] The terms "concrete," "suppositional" and "fantasy" are suggested as representative categories of test item content. In no study reviewed here did the author(s) use these and only these terms to distinguish different content of test items. Some authors, however, may have used one or more of these terms in classifying the content of their test items.

Ennis and Paulus, 1965). It seems reasonable to assume that some variation of test item content occurred in other studies, but was not reported. Variation in both the principle type and test item content would lend support to claims of mastery of logical principles, since we would normally be willing to credit a student with mastery of a logical principle only if he could apply such a principle in a variety of contexts. And certainly such variation of principle type and test item content must be used to justify any general claims about the successful teaching of "logic".

In general, the literature reviewed here suggests that secondary school students from diverse backgrounds can learn at least some principles of logical reasoning, and can do so under a wide variety of approaches. Several qualifications, however, must be added to this statement:

(1) Time spent teaching logic varies considerably among studies. It may be that studies finding no learning of some logical principles occurring would have found such learning to occur with a greater length of time spent on instruction.

(2) Notions of what constitutes "logic" vary widely and are often vaguely defined.

(3) Many experimenters developed their own criterion measures as well as serving as the instructor for the experimental group(s). Studies by these experimenters are open to charges of "teaching for the test," especially where highly significant differences in favor of the experimental group were found.

(4) Some claims of successful teaching of logic are based primarily upon subjective evaluations of pupil performance by the experimenter and/or classroom teacher.

Summary Table for Studies Cited in this Review

Author(s), Date, Student Grade	Logical Principles Taught[1]	Success Claimed?
Braxton, 1973, 7-9	detachment particular contraposition full transitivity full contraposition particular conversion particular inversion	Yes; significant differences in post-test score favoring experimental group found at each grade level
Carroll, 1970, 9	detachment particular contraposition particular conversion particular inversion	particular conversion only
Corley, 1959, 6-10	"intuition"[2] "inductive reasoning"[2] full transitivity detachment particular conversion of classes (Plus "demonstrative geometry"[2])	Assumed successful teaching of logic
Diffley, 1971, 7-9	particular inversion detachment particular conversion particular contraposition full transitivity full contraposition	Yes; highly significant differences found at 7th and 9th grade levels in favor of experimental groups on posttest score
Ennis and Paulus, 1965, 4-12	Most class and conditional principles (refer to Appendix for details)	Yes; significant differences favoring experimental groups found for many principles of class, conditional reasoning as well as total posttest score for grades 6, 11, 12.

Summary Table for Studies Cited in this Review

Experimental Design	Criterion Measure	Strengths	Weaknesses
Pretest/posttest and posttest only	"Cornell Conditional Reasoning Test, Form X"	N = 157; study included both valid and invalid forms; statistical analysis also compares socio-economic status and I.Q. as variables affecting performance on criterion; students were instructed by regular classroom teachers	
Posttest only/ two control groups	Experimenter designed; no name given	Deals with low math achievers; used different content-type in test items; valid and invalid forms included	Only six days of instruction for four principles
Posttest only/ survey-comparison	Experimenter-designed; no name given	Relatively large sample	Does not establish that logic was taught; only 5 days on logic instruction
Pretest/posttest control group	"Paulus Conditional Reasoning Test, Form Z"	N = 1542; regular teachers taught logic; both valid and invalid forms included; different content type in test items; students were "inner city"	
Pretest/posttest control group	"Cornell Class Reasoning Test" "Cornell Conditional Reasoning Test"	Extensive data analysis; wide range of subjects; both valid and invalid forms included; varied content type of test items	

Author(s), Date, Student Grade	Logical Principles Taught	Success Claimed?
Howell, 1967, 7	full transitivity detachment "and introduction"[2] "and elimination"[2] "not introduction"[2] "or elimination"[2]	Yes, significant differences favoring experimental groups found on posttest score
Hrabi, 1967, 7 & 9	"basic logical concepts"[2] "proof strategy"[2]	Assumed successful teaching of logic found to make significant difference in critical thinking ability at 9th grade level, but not 7th grade level
Hyram, 1957, 7-8 (grade level was not specified exactly)	"nature of thinking"[2] "tools of thinking"[2] "nature of definition"[2] "nature of eductive inference"[2] "nature of deductive inference"[2] "nature of experimentation"[2] "common errors in reasoning"[2]	Yes, significant differences favoring experimental group on posttest scores found
Karplus and Karplus, 1970, 5-Adult	None directly; second part of study used sample problem	Yes, scores on test showed substantial gains for group given example
Miller, E. H., 1955 (Grade level unspecified, presumably between 10-12)	Not stated	Yes, significant gains on posttest scores

Experimental Design	Criterion Measure	Strengths	Weaknesses
Posttest only/ control group; experiment replicated with same design	"Test of Inference Patterns"[3] "Inferential Reasoning Analysis Measure"[3]	N = 157; 15 42-minute periods spent on logic instruction	Report is not clear whether both valid and invalid forms included in instructional unit
Pretest/posttest control group for critical thinking measure; posttest only for logic measure	"Watson-Glaser Critical Thinking Appraisal, Form YM" "Logic Achievement Test"[3]	Regular teachers taught logic	Study does not establish that logic was successfully taught
Posttest only/ control group	Experimenter-designed; no name given	Groups closely matched on 4 variables; long period of logic instruction (4 months)	No level of significance reported in article; attributes experimental differences to teaching method
Example-non-example groups/ survey-comparison	"Islands Puzzle"[3]	N = 499; test responses analyzed for level of abstractness; justification for answer required	No statistical analysis; one problem used as test of logical ability
Pretest/posttest no control group	"Fallacy Recognition Test"[3]	Deals with large number of fallacies (29)	Only one day between pretest and posttest; article does not establish successful logic teaching; nature of instructional unit is unclear

Author(s), Date, Student Grade	Logical Principles Taught	Success Claimed?
Miller, W. A., 1969, 7	detachment particular contraposition full transitivity particular conversion particular inversion forward positive detachment (bi-conditionality)	Yes, based on teacher, author observations
Osborne, 1973, 7	No attempt made	Yes, significant differences favoring high-usage group found on test score
Retzer and Henderson, 1967, 7-8	"variable"[2] "open sentence"[2] "universal quantifier"[2] "universal set"[2] "universal statement (generalization)"[2] "instance of a generalization"[2] "counterinstance to a generalization"[2]	Yes, significant differences favoring experimental groups on posttest score found.
White, 1936, 7 (grade level was not specified exactly)	"conversion and opposition of propositions"[2] "deduction; syllogistic reasoning"[2] "induction"[2]	Yes, significant differences favoring experimental group found on 3 criterion measures

1. Whenever possible, logical principles are classified according to the system proposed by Ennis (1976).

2. These terms were quoted from the author's presentation.

3. This criterion measure was designed and named by the author.

4. The origin of this criterion measure is unclear.

Experimental Design	Criterion Measure	Strengths	Weaknesses
Pretest only	"Test of Logical Inference Patterns"[4]	Both valid and invalid forms included	No statistical analysis; no posttest; all subjective evaluations
High teacher use of conditional language vs. low teacher use; survey-comparison	"Cornell Conditional Reasoning Test, Level X"	Investigates influence of teacher verbal behavior, often overlooked	Raises many questions (see Appendix for details)
2 x 2 ANOVA; posttest only/ control group	Ratings of judges on verbalization ability	Investigates ability to form generalizations, often overlooked	Conclusions may not generalize to other nonmathematical contexts; article does not establish successful logic teaching
Posttest only/ control group	"Ballard's English Construction Test" "A composition" additional experimenter designed; no name given	Integrated logic and English instruction	Only one criterion measure clearly related to logical ability; no level of significance reported in article

REFERENCES

Braxton, L. M. The effects of instruction in sentential logic on the growth of the logical thinking abilities of junior high school students (Doctoral dissertation, University of Virginia, 1973). Dissertation Abstracts International, 1973, 34, 657A-658A. (University Microfilms No. 73-18,994)

Carroll, C. A. Low achievers' understanding of four logical inference forms: an analysis of difficulties and of the effect of instruction (Doctoral dissertation, Columbia University, 1970). Dissertation Abstracts International, 1970, 31, 4377A. (University Microfilms No. 71-06,151)

Corley, G. J. An experiment in teaching logical thinking and demonstrative geometry in grades six through ten (Doctoral dissertation, George Peabody College for Teachers, 1959). Dissertation Abstracts International, 1959, 24, 1375. (University Microfilms No. 59-03,492)

Diffley, W. J. Deductive reasoning in the inner city: a one hundred classroom experiment (Doctoral dissertation, University of Connecticut, 1971). Dissertation Abstracts International, 1971, 33, 188. (University Microfilms No. 72-14,226)

Ennis, R. H. An alternative to Piaget's conceptualization of logical competence. Child Development, 1976, 17, 903-919.

Ennis, R. H., & Paulus, D. H. Critical thinking readiness in grades 1-12 (phase I: deductive logic in adolescence) (U.S. Office of Education Publication No. 1680). Washington, D.C.: U.S. Government Printing Office, 1965. (ERIC Document Reproduction Service No. ED 003 818)

Howell, E. N. An experiment to improve the reasoning ability of seventh grade students (U.S. Office of Education Publication No. 6-8110). Washington, D.C.: U.S. Government Printing Office, 1967. (ERIC Document Reproduction Service No. ED 014 765)

Hrabi, J. S. An experimental study of a unit on logic and proof strategy in the grades seven and nine mathematics curriculum (Doctoral dissertation, University of Colorado, 1967). Dissertation Abstracts International, 1967, 28, 3528A. (University Microfilms No. 67-15,040)

Hyram, G. H. An experiment in developing critical thinking in children. Journal of Experimental Education, 1957, 26, 125-132.

Karplus, E. F. & Karplus, R. Intellectual development beyond elementary school I. deductive logic. School Science and Mathematics, 1970, 70, 398-406.

Linn, M C. & Thier, H. D. Effect of experimental science on development of logical thinking in children. Journal of Research in Science Teaching, 1975, 12, 49-62.

Miller, E. H. A study of difficulty levels of selected types of fallacies in reasoning and their relationships to the factors of sex, grade level, mental age, and scholastic standing. Journal of Educational Research, 1955, 49, 123-129.

Miller, E. H. A study of difficulty levels of selected types of fallacies in reasoning and their relationships to the factors of sex, grade level, mental age, and scholastic standing (Doctoral dissertation, Wayne University, 1952). Dissertation Abstracts International, 1952, 12, 852. (University Microfilms No. 00-04,313)

Miller, W. A. A unit in sentential logic for junior high school students; involving both valid and invalid inference patterns. School Science and Mathematics, 1969, 69, 548-552.

Osborne, A. R. Promoting logical ability. Theory Into Practice, 1973, 7, 286-291.

Retzer, K. A. & Henderson, K. B. Effect of teaching concepts of logic on verbalization of discovered mathematical generalizations. Mathematics Teacher, 1967, 60, 707-710.

White, E. E. A study of the possibility of improving habits of thought in school children by a training in logic. British Journal of Educational Psychology, 1936, 6, 267-273.

APPENDIX:

SUMMARIES AND REVIEWS OF CITED STUDIES

Braxton, L. M. The effects of instruction in sentential logic on the growth of the logical thinking abilities of junior high school students (Doctoral dissertation, University of Virginia, 1973). Dissertation Abstracts International, 1973, 34, 657A-658A. (University Microfilms No. 73-18,994)

Students in seventh, eighth, and ninth grade general mathematics classes were divided into three groups at each level, consisting of one experimental and two control groups. Experimental subjects received lessons in logic from their regular classroom teachers for 30 minutes for 10 days. These logical principles were taught:

(1) detachment
(2) particular contraposition
(3) full transitivity
(4) full contraposition
(5) particular conversion
(6) particular inversion

Control classes received no logic instruction. "The Cornell Conditional Reasoning Test, Form X" was used as a pretest and posttest for the experimental and one control group. The remaining control group received this test only as a posttest. This design was repeated for each grade level.

Among the results of the study are these:

(1) The experimental group at each grade level scored significantly higher on the posttest than the control groups ($p < .001$).

(2) When posttest scores for students at each grade level were compared, no significant differences were found among the mean scores of students at the three grade levels.

(3) Female students scored significantly higher than male students across grade levels on detachment ($p < .05$) and particular contraposition ($p < .01$).

(4) Braxton found this ordering of relative difficulty in mastering the six logical principles examined in the study (easiest to most difficult): full transitivity, full contraposition, detachment, particular contraposition, and particular conversion, particular inversion (equally difficult). Mastery was operationally defined by Braxton as correctly responding to at least five of the six test items corresponding to each logical pattern.

(5) Higher I.Q. students (I.Q. = 91 and above) performed significantly better than their lower I.Q. counterparts (I.Q. = below 91) as measured by total posttest score and scores on each of the valid logical principles.

Comments: The study appears well-designed and conducted. Braxton's criterion for mastery of a logical principle is clearly defined and fairly strict, which makes interpretation of results reported in the study more straightforward.

Carroll, C. A. Low achievers' understanding of four logical inference forms: an analysis of difficulties and of the effect of instruction (Doctoral dissertation, Columbia University, 1970). Dissertation Abstracts International, 1970, 31, 4377A. (University Microfilms No. 71-06,151)

Carroll worked with ninth grade parochial school students classified as low achievers in mathematics by scores on a high school entrance examination. "Low achievers" scored below the 15th percentile in mathematics. However, in order to participate in the study, subjects must have scored above the 25th percentile in reading.

The study involved two schools, one for girls and the other for boys. At the girls school, two experimental groups of twelve subjects received instruction in conditional reasoning principles; one control group of twelve subjects received instruction in probability; and another control group of 37 students received no instruction. At the boys school, two experimental groups of twelve subjects received instruction in principles of conditional reasoning; one control group of twelve subjects received instruction in probability; and another control group of 84 subjects received no instruction. All control group subjects were also classified as low achievers in mathematics.

Carroll developed the instructional materials and served as the instructor for both the experimental groups and the control groups receiving instruction in probability. Instruction consisted of one 40-minute period per week for six weeks. The following conditional reasoning principles were taught:

(1) detachment
(2) particular contraposition
(3) particular inversion
(4) particular conversion

Carroll developed two logically equivalent tests of conditional reasoning, one of which was administered as a pretest and the other as a posttest. The tests measured conditional reasoning ability only on the above four principles. Content of the test items was classified as:

(1) familiar
(2) symbolic
(3) misleading
(4) removed from reality

In addition, the "Matching Familiar Figures Test" of reflective-impulsive cognitive style, developed by Jerome Kagan, was administered to all experimental groups at the conclusion of instruction.

Among the results of the study were these:

(1) The percentage of experimental group subjects who improved on posttest score did not differ significantly from the percentage of control group subjects who improved on this measure. "Improvement"

is defined as obtaining a higher score on posttest than pretest. Carroll declined to compare mean scores, as she argues that means can be inflated or deflated by a few extreme scores, and therefore percent of improvement is a more reasonable measure of any learning which occurred.

(2) The percentage of experimental subjects who improved on correctly applying the principle of particular conversion was significantly higher than the corresponding percentage of their control group counterparts ($p < .01$). No other significant differences for application of principles were found.

(3) Improvement of experimental subjects was significantly better than the improvement of control group subjects on items classified as "misleading" and "removed from reality." ($p < .01$).

(4) Generally, subjects performed best on applying the valid conditional reasoning principles (detachment and particular contraposition) and worst on identifying invalid principles (particular inversion and particular conversion).

Comments: Carroll provided relatively little instruction ("six lessons") in her four logical principles. Additional significant differences may have been found if the duration of instruction had been increased. The study is noteworthy in that although subjects were all low achievers in mathematics, the experimental groups learned to apply successfully the principle of particular conversion at the conclusion of instruction. This same principle was found to be one of the most difficult for control group subjects to apply correctly.

Carroll's criterion for "improvement" seems questionable; presumably, a 1-point increase in total score from pretest to posttest qualifies as "improvement." Such small differences are probably better attributed to measurement error or practice effects. Carroll also states: "A considerable portion of each group showed improvement; at least 68 percent of every group had higher scores on the posttest than on the pretest" (p. 70-71). Since improvement was evidenced for every group, even those which received no instruction, it is not clear that improvement on posttest score can be attributed to successful logic teaching.

Corley, G. J. An experiment in teaching logical thinking and demonstrative geometry in grades six through ten (Doctoral dissertation, George Peabody College for Teachers, 1959). Dissertation Abstracts International, 1959, 24, 1975. (University Microfilms No. 59-03,492)

In the first part of the study, Corley instructed one group of students at each grade level of grades 6-10 in "the three methods of reaching general conclusions: intuition, inductive reasoning, and deductive reasoning" (N = 220). By "deductive reasoning" Corley means the following class-reasoning principles:

(1) full transitivity
(2) detachment
(3) particular conversion of classes* (--an invalid move):

all A are B
all C are B
---
therefore, all A are C

Instruction lasted for five classroom periods of 55 minutes each (one week). At the conclusion of instruction, students were administered a reasoning test developed by the experimenter, and mean scores for each grade level were compared. The reasoning test was designed to measure: (1) "knowledge of the three methods of reaching conclusions, especially ability to distinguish between them; (2) ability to detect the validity or invalidity of a syllogism; (3) familiarity with the parts of a syllogism; and (4) ability to diagram a syllogism" (p. 41). Areas 1 and 2 received the most emphasis in the test (14 questions out of 18 total questions).

In the second part of his study, Corley instructed one group of students at each grade level of grades 6, 7, 8 and 10 in general reasoning and demonstrative geometry (N = 98). Instruction in reasoning was identical to that described earlier. Demonstrative geometry instruction lasted four school weeks at 55 minutes per day. At the conclusion of instruction, subjects were given a test developed by Corley which was designed to measure: (1) knowledge of geometric terms and concepts; (2) understanding of proofs and geometric theorems; and (3) ability to derive valid conclusions from premises (syllogistic reasoning). Mean and part scores for each grade level were compared.

Among the findings for the first part of the study were these:

(1) ANOVA showed grade level and I.Q. to be significant factors in the design ($p < .05$). No significant grade level x I.Q. interaction was found. Multiple comparisons using t-tests showed that sixth grade students scored significantly lower on the general reasoning test than students at each of the higher grades ($p < .05$).

* This principle is not mentioned by Ennis (1976); hence the classification is my own.

Grade 9 students also scored significantly higher than grade 7 and grade 8 students on the general reasoning test ($p < .05$). No other significant differences in scores between grade levels were found.

(2) Students with I.Q.'s of above 107.5 scored significantly higher on the general reasoning test than students with I.Q.'s of 92.5 to 107.5 ($p < .05$). I.Q. was measured by the "Lorge-Thorndike Nonverbal Intelligence Test, Level 4."

The second part of the study included these findings:

(1) ANOVA showed grade level and I.Q. to be significant factors in the design ($p < .05$). No significant grade level x I.Q. interaction was found.

(2) Multiple comparisons using t-tests showed that on part 3 and total score of the geometry and reasoning test, grade 6 students scored significantly lower than students in each of grades 7, 8, and 10 ($p < .05$). Part 3 tests the ability to derive valid conclusions from premises (syllogistic reasoning).

Comments: Corley used students in a parochial school and a "public school for white pupils" (p. 37-38). He argues that "there was no apparent reason to assume that there was any difference between students of this school [the private school] and students of corresponding grades in the public schools so far as this study was concerned" (p. 38). He does not address the question of whether students in the public school for white pupils constitute a representative sample from a normal population.

In addition, Corley's study does not establish that any teaching of logic occurred, since no control groups were used as comparisons at each grade level. Corley, however, appears willing to assume that he successfully taught logic.

Diffley, W. J. Deductive reasoning in the inner city: a one hundred classroom experiment (Doctoral dissertation, University of Connecticut, 1971). Dissertation Abstracts International, 1971, 33, 188. (University Microfilms No. 72-14,226)

One hundred classes of seventh, eighth and ninth graders were divided into experimental and control groups. The experimental group received "quasi-programmed" instruction for ten 50-minute periods from their regular teachers in six logical principles:

(1) particular inversion
(2) detachment
(3) particular conversion
(4) particular contraposition
(5) full transitivity
(6) full contraposition

The control group received no instruction in any logical principles.

The "Paulus Conditional Reasoning Test, Form E (Assessing)" was used as a pretest and posttest for both groups. Posttest results showed the experimental group to have scored significantly higher than the control group ($p < .005$). The study also found invalidity principles more difficult to master than validity principles. Posttest comparisons of classes at the eighth grade level showed non-significant differences in test performance. The direction of this difference was unspecified, and amounted to .01 points.

Comments: Diffley's study is noteworthy in that he deals with a relatively large population of minority students (predominantly Black American and Puerto Rican). The highly significant difference found in favor of the experimental group and the fact that students were taught by their regular classroom teachers suggest that ability to master principles of deductive logic is not limited to students with a particular cultural background. In general, this study is well designed and executed; the results also receive careful analysis.

Ennis, R. H. & Paulus, D. H. Critical thinking readiness in grades 1-12 (phase I: deductive logic in adolescence) (U.S. Office of Education Publication No. 1680). Washington, D.C.: U.S. Government Printing Office, 1965. (ERIC Document Reproduction Service No. ED 003 818)

The authors administered the "Cornell Class Reasoning Test" to one class of students in grades 4, 6, 8, 10 and 12 and the "Cornell Conditional Reasoning Test" to one class each of students in grades 5, 7, 9 and 11. Class size ranged from 20-30 students. These experimental groups received instruction from the project's staff in class or conditional logic for 15 instructional days, for approximately 40 to 50 minutes per day. Control groups from both the same school system and a nearby school system were administered the same tests, but received no logic instruction. The experimental groups receiving instruction in conditional logic were taught the following principles:

(1) detachment
(2) particular inversion
(3) particular conversion
(4) particular contraposition
(5) full transitivity
(6) full contraposition
(7) full conversion
(8) reverse positive detachment (biconditionality)
(9) additional 'only if' principles

The experimental groups receiving instruction in class logic were taught the following principles:

(1) full transitivity
(2) full conversion
*(3) symmetry of class exclusion (No A's are B's. Therefore, no B's are A's).
*(4) exclusive transitivity (All A's are B's; no B's are C's; therefore, no A's are C's).
(5) particular inversion
(6) additional relational principles of class logic

About six weeks after the conclusion of instruction, a posttest was administered to all subjects (subjects took the same posttest as pretest). Analysis of the results showed that the grade 11 experimental group did significantly better on most of the conditional reasoning principles than control group subjects (.05 level). The grade 11 experimental group also scored significantly higher on total posttest score than subjects in the control group. Experimental groups at lower grade levels showed significantly higher posttest scores only on isolated conditional reasoning principles. For the groups taught class reasoning principles, their total posttest scores were significantly higher than scores of control group subjects at grades 6 and 12 (.05 level); experimental groups at grades 6, 8 and 12 showed significantly higher scores on some class reasoning principles than their control group counterparts (.05 level). In no case did the control group score significantly higher than the experimental group for groups taught class reasoning principles.

* These terms are my own; the authors of this study do not use any label for these principles.

Note: The report contains extensive analysis of the results, including ANCOVA. The control group at the 9th grade level scored significantly higher than the experimental group both on total posttest score and several conditional reasoning principles. The authors suggest several explanations for this result.

Comments: This study is probably the most comprehensive one cited in this review. The study is carefully designed and analyzed.

Howell, E. N. An experiment to improve the reasoning ability of seventh grade students (U.S. Office of Education Publication No. 6-8110). Washington, D.C.: U.S. Government Printing Office, 1967. (ERIC Document Reproduction Service No. ED 014 765)

In collaboration with other individuals, Howell prepared a text entitled, Effective Thinking for use in a logic course at a Wisconsin junior high school. A total of 157 subjects participated in the experiment, which consisted of two parts. Each part is reported separately below.

Students were assigned a composite score of performance on three variables (score on "Otis Quick-Scoring Mental Ability Test, Form A" given in the 4th grade, score on "Iowa Tests on Basic Skills," given during the last month of 6th grade, and score on a five-point scale of 6th grade classroom performance as evaluated by teacher). The students were then rank ordered and divided into blocks of four. Within each block, students were randomly assigned to either of two experimental or two control groups (four groups total, two of each).

Part I:

One experimental group received logic instruction for the entire first quarter of the school year from their regular teachers. Fifteen 42-minute class periods (approximately three weeks) were spent on deductive logic, both valid and invalid forms. These forms included:

(1) "and introduction"
P.
Q.
P and Q

(2) "and elimination"
P and Q
P

(3) "or introduction"
P.
P or Q

(4) "not introduction"
if P then Q
if P then -Q
-P

(5) full transitivity

(6) detachment

(7) particular contraposition

The control group was given a course in speech. At the end of the quarter, both groups were given the "Test of Inference Patterns," developed by Howell. The experimental group was found to score significantly higher than the control group on total test score ($p < .05$).

Part II:

In the next quarter, the second experimental group received the same logic instruction, while the second control group received instruction in Art. At the end of the quarter, both groups were administered the "Inferential Reasoning Analysis Measure," also developed by Howell. Differences in total test score between the groups were found to be significant and in favor of the experimental group ($p < .05$).

Among conclusions which emerge from a statistical analysis of the data are these:

(1) Significant sex differences in total test scores favoring female students were found on the "Inferential Reasoning Analysis Measure" ($p < .05$); no such differences were found on the other measure.

(2) No significant interaction between treatment and sex was observed.

(3) Generally, higher I.Q. students tended to profit more from instruction than lower I.Q. students.

Comments: This study is generally well designed and executed. The results are given ample statistical analysis. However, Howell does not make it clear whether logic instruction considered invalid forms. The "Test of Inference Patterns" required students to identify invalid forms. The "Inferential Reasoning Analysis Measure" did not.

Hrabi, J. S. An experimental study of a unit on logic and proof strategy in the grades seven and nine mathematics curriculum (Doctoral dissertation, University of Colorado, 1967). Dissertation Abstracts International, 1967, 28, 3528A. (University Microfilms No. 67-15,040)

Three grade 9 algebra classes and two grade 7 mathematics classes were given six weeks of instruction in basic logical concepts and proof strategy from their regular teachers. Control groups received no such instruction; each teacher taught both an experimental and a control group. The amount of time spent on instruction varied from a low of approximately 32 minutes per day to a high of approximately 43 minutes per day. The amount of homework assigned also varied among teachers, from a low of approximately 9 minutes per day to a high of approximately 26 minutes per day.

The instructional materials consisted of the first two chapters of A First Course in Mathematical Logic by Patrick Suppes and Shirley Hill; the article, "Logic: For Teacher, for Pupil" by R. L. Swain; and additional materials created by the experimenter. Generally, students were introduced to the logical operators "or"; "and"; and "if...then"; logical inference; and symbolic logic.

Students were administered the "Lorge-Thorndike Intelligence Test, Level 4, Form A"; the "Iowa Test of Basic Skills, Form 4"; and the "Iowa Test of Educational Development, Form X-4" at the beginning of instruction. The "Watson-Glaser Critical Thinking Appraisal, Form YM" was used as a pretest and posttest for all groups. In addition, Hrabi developed a "Logic Achievement Test" which was given to all groups as a posttest. The test was based heavily on a test developed by Patrick Suppes and Frederick Binford described in the article: "Experimental Teaching of Mathematical Logic in Elementary School," The Arithmetic Teacher, Volume 12 (1965) p. 187-196. The "Watson-Glaser Critical Thinking Appraisal, Form YM" was also administered to all groups two months after the conclusion of instruction as a retention measure. Teacher responses to a questionnaire regarding their reactions to the logic unit were also included in the study.

Among the findings in ANOVA and ANCOVA are these:

(1) No significant differences in critical thinking ability were found between experimental and control groups at the grade seven level on the posttest measure. Grade nine experimental students scored significantly higher than their control group counterparts on this measure ($p < .05$), and the experimental group was still significantly superior when both groups were re-tested 2 months later ($p < .05$).

(2) Significant interaction on the critical thinking measure was found between treatment and sex at the grade nine level ($p < .01$); female experimental students achieved higher total posttest scores than female control students. Male students showed the opposite relationship.

(3) Grade nine students achieved significantly better mean scores on the "Logic Achievement Test" than grade seven students'when mean scores were adjusted for differences in intelligence ($p < .01$).

Comments: Hrabi made no comparison of experimental and control group subjects' scores on the "Logic Achievement Test." Hence scores on this measure cannot be used to justify the claim that logic was successfully taught at a particular grade level.

On the assumption that the "Watson-Glaser Critical Thinking Appraisal, Form YM" measures logical ability to some extent, Hrabi's study suggests that experimental grade nine students were successfully taught logic, since grade nine experimental group students scored significantly higher than their control group counterparts on this measure ($p < .05$). Posttest scores of grade seven experimental group students did not differ significantly from posttest scores of grade seven control group students on the critical thinking measure. The study therefore presents no information which establishes that grade seven experimental group subjects were taught logic.

In addition, interpretation of results is complicated by the fact that neither teaching time nor amount of homework (and presumably type of homework) assigned were controlled. Hrabi also notes that one grade nine experimental group, one grade nine control group, and one grade seven experimental group were composed of "selected high ability" students (p. 11).

The study provides extensive statistical analysis of the data.

Hyram, G. H. An experiment in developing critical thinking in children. Journal of Experimental Education, 1957, 26, 125-132.

An experimental group of 33 children in the "upper grades of three public elementary schools" (I judge this to be approximately grades 7 and 8) with an I.Q. of 110 or better were taught 7 concepts of logical thinking by the experimenter. Instruction lasted for 4 months at 50 min./day. These concepts included:

(1) nature of thinking
(2) tools of thinking
(3) nature of definition
(4) nature of eductive inference
(5) nature of deductive inference
(6) nature of experimentation
(7) common errors in reasoning

Hyram does not define "eductive inference" in the article.

At the completion of instruction, performance of the experimental group on a test of reasoning ability (created by the experimenter) was compared with performance of a control group of 33 students matched in 5 variables, and was found to differ significantly in favor of the experimental group. Hyram attributes this difference to method of instruction (Socratic questioning) although he states that the control group received no instruction in the 7 areas listed above.

Comments: Although Hyram attributes the significantly higher posttest scores of the experimental group to "method of instruction," this seems misleading. He argues that "method of instruction" was "...the only known variable of importance..." and that "...this factor seemed to be a cause sufficient to produce the desired growth in pupil logicality" (p. 129). Hyram's study suggests that logic can be taught successfully at this level. It seems doubtful, however, that the study can sustain the claim that the Socratic method of instruction produced the significant differences between experimental and control groups. The fact that the experimental group received instruction in logic whereas the control group did not would seem to be a more likely explanation for the subsequent significant difference found in the study. No level of significance is reported in the article.

Karplus, E. F. & Karplus, R. Intellectual development beyond elementary school I. deductive logic. School Science and Mathematics, 1970, 70, 398-406.

The experimenters gave the "Islands Puzzle"[1] which they created to 449 subjects divided into 6 groups. Subjects ranged from suburban 5th and 6th graders to members of the American Association of Physics Teachers. Responses to the problem, which was designed as a measure of abstract reasoning ability (actually, conditional reasoning ability), and their explanations or justifications were recorded. Median number of correct answers showed an increase across groups up to the high school age level, at which it stabilized. The same pattern was characteristic of the level of abstraction of the justification or explanation given.

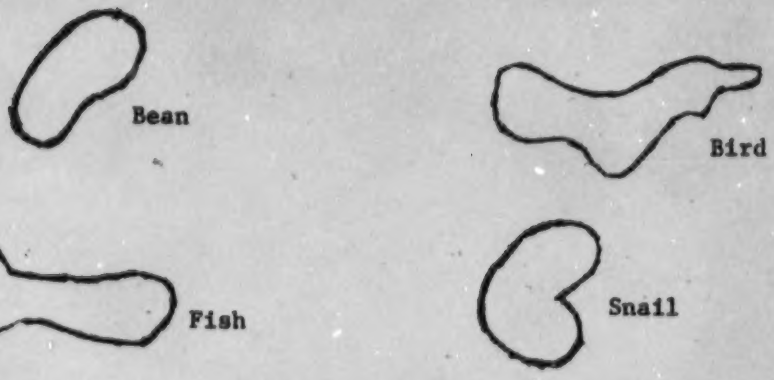
However, when similar subjects in grades 5-12 were given a sample problem similar to the "Islands Puzzle" before being tested, their scores showed substantial gains in both number of correct answers and level of abstraction in explanation compared to the no-practice subjects in the same age groups (no statistical analysis). The study suggests that the use of examples may facilitate learning of logical reasoning patterns.

Note: Although subjects with "practice" performed better, the experimenters note that subjects' reactions to the example puzzle were "subdued but noticeable signs of dismay."

Comments: The authors provide no rationale for accepting the "Islands Puzzle" as a measure of logical ability. Since the second part of the study dealt only with one example, it only suggests that examples may facilitate learning of logical principles. The second part of the study also suggests the benefit of practice in learning logical principles. However, the study does not establish either of these conclusions.

[1] A copy of the "Islands Puzzle" appears on the next page.

Islands Puzzle



Introduction: The puzzle is about four islands in the ocean. People have been traveling among these islands by boat for many years, but recently an airline started in business. Listen carefully to the clues I give you about possible plane trips. The trips may be direct or they may include stops on one of the islands. When I say a trip is possible, it can be made in both directions between the islands.

This is a map with the four islands, called Bean Island, Bird Island, Fish Island, and Snail Island. You may make notes or marks on your map to help you remember the clues. Raise your hand if you have questions about the clues.

First clue: People can go by plane between Bean and Fish Island.

Second clue: People cannot go by plane between Bird and Snail Islands. Use these two clues to answer question 1.

Question 1: Can people go by plane between Bean and Bird Islands?
Yes_____ No_____ Can't tell from the two clues._____
Explain your answer.

Third clue: People can go by plane between Bean and Bird Islands. Use all three clues to answer questions 2 and 3. Don't change your answer to question 1.

Question 2: Can people go by plane between Fish and Bird Islands?
Yes_____ No_____ Can't tell from the three clues._____
Explain your answer.

Question 3: Can people go by plane between Fish and Snail Islands?
Yes_____ No_____ Can't tell from the three clues._____
Explain your answer.

This puzzle was reproduced exactly as it appeared in the original article. Only the figures were redrawn.

Miller, E. H. A study of difficulty levels of selected types of fallacies in reasoning and their relationships to the factors of sex, grade level, mental age, and scholastic standing. Journal of Educational Research, 1955, 49, 123-129.

Miller developed two parallel forms of a "Fallacy Recognition Test" each consisting of 50 items. Three items were "warm up"; 18 items were distractors and were not scored; the remaining 29 items were "examples of fallacies in reasoning as selected from over one hundred books on logic, and edited with respect to reading level and content material to fit the high school population surveyed" (p. 123). Miller's "Fallacy Recognition Test" includes examples of 29 fallacies listed in the article; presumably, each of the scored 29 test items exemplified a single fallacy. Several of the fallacies are principles of deductive logic or syllogistic reasoning, including denying the antecedent (particular inversion), affirming the consequent (particular conversion), and undistributed middle term.

An experimental group of 32 high school students (the article does not specify the grade level) was tested with one form of the "Fallacy Recognition Test." The following day, the experimental group received one 40-minute class period of "general instruction covering some of the types of reasoning as found in the 'Fallacy Recognition Test'" (p. 126-127). The experimental group students were retested the following day using the other form of the "Fallacy Recognition Test." Posttest scores were found to be significantly higher than pretest scores ($p < .005$).

Comments: Miller reports no experimental-control group comparisons in this study, although 297 grade 10, 11, and 12 students were administered the "Fallacy Recognition Test" for other purposes. A comparison of the scores of this group with the scores of the experimental group would provide some evidence that successful instruction had occurred. The article provides no evidence that posttest gains can be attributed to successful instruction. Further, the nature of the instruction is unclear; Miller may or may not have taught logic in the 40 minutes of instruction given the experimental group. The short period between pretest and posttest also suggests that any improvement in reasoning ability which occurred is likely to be temporary. Such improvement could also be attributed to practice effects.

This article is based on Miller's dissertation research. Presumably, the dissertation provides more complete information than is found in the article.

Miller, W. A. A unit in sentential logic for junior high school students; involving both valid and invalid inference patterns. School Science and Mathematics, 1969, 69, 548-552.

The author taught seventh grade students 6 logical inference forms in 12 50-minute class periods, covering the period of April 15-May 28, using materials he created. Students were given the "Test of Logical Inference Patterns" as a pretest; no posttest was administered.

Miller taught these 6 principles:

(1) detachment
(2) particular contraposition
(3) full transitivity
(4) particular conversion
(5) particular inversion
(6) forward positive detachment (biconditionality)

Miller's conclusions included these:

(1) "At the conclusion of the unit, the students were able to correctly test the validity or invalidity of an inference pattern. Included were patterns which the students had not seen before."

(2) "Even though a statistical evaluation was not made, it seems safe to say that the students did learn and enjoy this new material."

These conclusions are based entirely upon "subjective observations of the regular teacher and researcher."

Comments: According to pretest results students had already achieved mastery as defined by Miller (13 or greater items correct out of 20) on 3 of 4 valid principles (forward positive detachment had not been learned); no students had mastered the invalidity principles. No statistical analysis of the data was performed. In addition, the experimental design is incomplete because no posttest was given. "Subjective evaluations" are probably inappropriate as the sole basis for conclusions drawn about students' logical abilities and mastery of logical principles. Miller also accepts the traditional interpretation of material implication.

Osborne, A. R. Promoting logical ability. Theory Into Practice, 1973, 7, 286-291.

Osborne investigated the effects of teacher use of conditional language on the conditional reasoning abilities of seventh grade students in mathematics classes. Students in 20 mathematics classes were given the "Cornell Conditional Reasoning Test, Form X," as a pretest. Osborne videotaped 5 class sessions of each of the 20 math teachers throughout the length of the study (September-January), and trained analysts determined the frequency of usage of language of conditional logic by the teacher. The "Cornell Conditional Reasoning Test, Form X" was administered as a posttest. Posttest scores for students in classes of the top five users were compared with scores of students in the classes of the lowest five users. A significant difference favoring students in the high-usage classes was found ($p < .03$). Although no overt attempts to teach deductive logic were made, the study suggests that learning of logical operations may be greatly influenced by teacher verbal behavior. The study also analyzed the data in terms of the content classification of test items (concrete familiar, suggestive, and negative).

Comments: The study raises several questions. For example, no account was taken of the correctness of the conditional reasoning used by the classroom teacher. It does not seem reasonable to suppose that simple exposure to conditional statements (regardless of their correctness) is sufficient to improve conditional reasoning ability. Further, no note is taken of the degree of classroom interaction between student and teacher, or the use of conditional statements by students. It might be the case that students in high-usage classes had a greater opportunity to reason conditionally (and hence use conditional language and acquire greater familiarity with valid principles) than students in low-usage classes. Finally, although the study reports the classes selected for study were chosen randomly, it does not state that the within-class distribution of students was also random. It might be that higher-ability students select the "good" teachers; these "good" teachers might also be the ones who make the most use of conditional language in class.

Retzer, K. A. & Henderson, K. B. Effect of teaching concepts of logic on verbalization of discovered mathematical generalizations. Mathematics Teacher, 1967, 60, 707-710.

The authors studied 80 junior high school students (37 male, 43 female) in the seventh and eighth grades. The 80 students were partitioned into a 2 x 2 design. Factor A was the study of logical concepts; factor B was ability level. The two levels of factor A were completion of the unit, Sentences of Logic and not having completed the unit. The duration of the study of Sentences of Logic is not reported in the article. The two levels of factor B were "gifted" (I.Q. of 135 and above) and "college capable but not gifted" (I.Q. of 116-128). The unit, Sentences of Logic, dealt with such concepts as:

(1) variable
(2) open sentence
(3) universal quantifier
(4) universal set
(5) universal statement (generalization)
(6) instance of a generalization
(7) counterinstance to a generalization

The study investigated whether completion of the unit Sentences of Logic had any effect on ability to verbalize discovered mathematical generalizations.

Both groups were led to discover generalizations about vectors from a programmed instruction unit. The unit included hints on formulating precise generalizations. Students were scored by a panel of 3 mathematics teachers and received verbalization scores on 2 scales:

(1) strict: required the student to utter a complete, correct declarative sentence.
(2) lenient: allowed student to give technically incorrect statements acceptable to "non-meticulous" mathematicians.

Subjects who studied Sentences of Logic did significantly better than those who did not at verbalizing generalizations ($p < .005$). Gifted students verbalized significantly better than non-gifted ($p < .005$). Interaction of main effects was significant at .05 level for strict standard, .10 level for lenient standard.

Comments: Although the authors claim a significant difference in verbalization scores, the article fails to identify which scale was used in determining this difference. As the article is written, it is impossible to determine if students who completed the logic unit differed significantly from their counterparts in verbalization scores on the strict standard, the lenient standard, both, or some combination.

The authors assume that ability to verbalize mathematical generalizations is a measure of logical ability. Without this assumption, the results of the study do not establish that any teaching of logic occurred.

It is also difficult to know if this study reports generalizable results. We might wish to improve students' abilities to formulate precise generalizations. However, in many fields the generalizations we are forced to live with are much less precise than in mathematics; often it is not clear that a particular instance represents an example of a general principle, or if it does, the question of which principle it exemplifies is open to argument.

White, E. E. A study of the possibility of improving habits of thought in school children by a training in logic. British Journal of Educational Psychology, 1936, 6, 267-273.

White used two classes of boys, each with a mean age of nearly 13 years. The experimental class received instruction in logic for one hour per week for 3 months in addition to their regular grammar instruction. The control group received only their grammar instruction. White's instruction in logic included these areas, among others:

(1) "conversion and opposition of propositions"
(2) deduction; syllogistic reasoning
(3) induction

At the end of three months, both groups were tested using three tests:

(1) "a reasoning test"
(2) Ballard's English Construction Test
(3) a composition to write

Analysis of the results showed that the experimental group mean differed significantly from the control group mean for each test. The difference always favored the experimental group. The difference was most significant for mean scores on the reasoning test (no level of significance reported in article).

Note: Exercises used in the study can be found in Thought and Expression, also by E. E. White (London, Edward Arnold and Co.)

Comments: Although White claims significant differences, no level of significance is reported. The nature of the "reasoning test" used is hinted at in the article, but a complete description is not provided. Three aspects of the reasoning test appear to be related to deductive logic; the other one involves problems of classification (determining which item in a series does not "belong"). The article provides no information on White's second and third criterion measures.